

# THE METHOD TO DETERMINE THE $\mathcal{CP}$ NATURE OF HIGGS BOSONS FROM DECAYS TO TAU LEPTONS AT LC \*

MAŁGORZATA WOREK

Institute of Nuclear Physics of the Polish Academy of Sciences  
Radzikowskiego 152, 31-342 Krakow, Poland.

and

Institute of Physics, University of Silesia  
Uniwersytecka 4, 40-007 Katowice, Poland.

e-mail: Malgorzata.Worek@ifj.edu.pl

We demonstrate how the transverse  $\tau^+\tau^-$  spin correlations can be used to determine whether a decaying Higgs boson is a mixed  $\mathcal{CP}$  eigenstate, thereby directly probe the presence of  $\mathcal{CP}$  violation in the neutral Higgs boson sector. We investigate the subsequent decay chain  $H \rightarrow \tau^+\tau^- \rightarrow \rho^+\bar{\nu}_\tau\rho^-\nu_\tau \rightarrow \pi^+\pi^0\bar{\nu}_\tau\pi^-\pi^0\nu_\tau$ . The prospects for the measurement of the pseudoscalar admixture in the  $H\tau^+\tau^-$  coupling to a Standard Model Higgs boson with a mass of 120  $GeV$  are quantified for the case of  $e^+e^-$  collisions at 350  $GeV$  center-of-mass energy and 1  $ab^{-1}$  integrated luminosity. The Standard Model Higgsstrahlung production process is used as an example.

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## 1. Introduction

A most distinctive feature of the extended models of Standard Model ( $\mathcal{SM}$ ) such as Minimal Supersymmetric Standard Model ( $\mathcal{MSSM}$ ), or general Two Higgs Boson Doublet Model ( $2\mathcal{HDM}$ ), is the existence of additional Higgs bosons, *i.e.* a charged Higgs boson pair,  $H^\pm$ , one  $\mathcal{CP}$  odd scalar  $A^0$  and two  $\mathcal{CP}$  even scalars  $h^0, H^0$ . Their mass and coupling patterns vary with the model parameters. The discovery of a neutral Higgs boson with mass in the range  $114\text{ GeV} < M_H < 140\text{ GeV}$  will raise the question whether the observed particle is the  $\mathcal{SM}$  Higgs boson or the lightest boson from the Higgs boson sector of a  $\mathcal{SM}$  extension. Even if the one doublet

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$\mathcal{SM}$  turns out to be a good model, we will want to strictly determine that the observed Higgs boson is indeed  $\mathcal{CP}$  even in its nature.

Whichever scenario is realized in Nature, the major goal of future high energy experiments will be to distinguish among these models. The precision measurements of all relevant Higgs boson couplings are powerful to obtain information about additional Higgs doublets, their structure and masses. Such measurements can only be performed at the  $e^+e^-$  linear collider ( $LC$ ) such as *TESLA*. The determination of the quantum numbers of the Higgs boson(s) and sensitivity to the  $\mathcal{CP}$  violation represent a crucial part of this test and will allow to establish the Higgs boson mechanism as the mechanism of the electroweak symmetry breaking.

The determination of the parity and the parity mixing of spinless Higgs bosons have been extensively investigated in Refs. [1–8]. The model independent identification of parity of the Higgs particle has recently been demonstrated for  $H/A^0 \rightarrow \tau^+\tau^+$  decay chain in Refs. [9–13].

In Ref. [9], see also Ref. [12] the reaction chain  $e^+e^- \rightarrow Z(H/A^0)$ ,  $H/A^0 \rightarrow \tau^+\tau^-$ ,  $\tau^\pm \rightarrow \pi^\pm \bar{\nu}_\tau(\nu_\tau)$  was studied. It was found that even small effects of smearing seriously deteriorate the measurement resolution. However, the spin effects of the decay chain  $H/A^0 \rightarrow \tau^+\tau^- \rightarrow \rho^+ \bar{\nu}_\tau \rho^- \nu_\tau \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau \pi^- \pi^0 \nu_\tau$  give a parity test independent of both model (*e.g.*  $\mathcal{SM}$ ,  $\mathcal{MSSM}$ ) and Higgs boson production mechanism (*e.g.* Higgsstrahlung, WW fusion). Using reasonable assumptions about the  $\mathcal{SM}$  production cross section and about the measurement resolutions we have found that, with  $500 \text{ fb}^{-1}$  of luminosity at a  $500 \text{ GeV}$   $e^+e^-$  linear collider, the  $\mathcal{CP}$  of a  $120 \text{ GeV}$  Higgs boson can be measured to a confidence level greater than 95%, see Ref. [10].

In Ref. [11] we demonstrated that a measurement of the  $\tau$  impact parameter in one-prong  $\tau$  decay is useful for the determination of the Higgs boson parity in the same decay chain. For a detection set-up such as *TESLA*, use of the information from the  $\tau$  impact parameter can improve the significance of the measurement of the parity of a Standard Model  $120 \text{ GeV}$  Higgs boson to  $\sim 4.5\sigma$ .

In this paper we continue to investigate the case of a Higgs boson decay into  $\tau^+\tau^-$  and we concentrate on the more general case where mixed scalar and pseudoscalar couplings of the Higgs boson to  $\tau$  leptons are simultaneously allowed, see Ref. [13] for details study.

The rest of the paper is organized as follows. In Section 2 we present general information about measurement of the  $\mathcal{CP}$  quantum numbers of Higgs boson using  $\tau\tau$  decay channel. In section 3 we recall basic properties of the density matrix for the pair of  $\tau$  leptons produced in Higgs boson decay. In Section 4 we define an observable. Numerical results are presented in Section 5. The Summary closes the paper.

## 2. $\mathcal{CP}$ properties of the Higgs boson from $\tau\tau$ decay

Various more or less complicated extensions of the  $\mathcal{SM}$  have been proposed in literature, see *e.g.* Ref. [14–20], but their common feature is that in case of models with  $\mathcal{CP}$  violation in the Higgs boson sector the mass eigenstates of the neutral Higgs bosons are not precisely  $\mathcal{CP}$  eigenstates  $h^0$ ,  $H^0$  and  $A^0$ .  $\mathcal{CP}$  violation results in three neutral states of mixed  $\mathcal{CP}$  character. For these models to estimate precision of the  $H - A^0$  mixing angle is essential. We consider a potential of establishing  $\mathcal{CP}$  properties of Higgs bosons, without any assumption about a  $\mathcal{CP}$  violation model, from the analysis of the angular distributions of the  $\tau^+\tau^-$  decay products in the plane transverse to the  $\tau^+\tau^-$  axes. The transverse spin effects in  $\tau$  pair production, reflected in correlations between  $\tau$  decay products, are helpful to distinguish between the scalar  $\mathcal{J}^{\mathcal{PC}} = 0^{++}$ , pseudoscalar  $\mathcal{J}^{\mathcal{PC}} = 0^{-+}$  and mixing natures of the spin zero (Higgs) particles. When the Higgs boson is light enough that the  $W^+W^-$  decay channel remains closed, the most promising decay channel of  $\mathcal{SM}$  neutral Higgs particle sensitive for spin correlations, is the  $\tau^+\tau^-$  mode. The  $\tau^+\tau^-$  channel is useful in the  $\mathcal{SM}$  for Higgs masses less than  $\sim 140$  *GeV*. Up to this mass, the Higgs particle is very narrow,  $\Gamma(H_{SM}) \leq 10$  *MeV* with  $\mathcal{BR}(H_{SM} \rightarrow \tau^+\tau^-) \sim 9\%$ . In Supersymmetric theories, the  $\tau^+\tau^-$  channel is useful over a much larger mass range. In this approach the  $\mathcal{CP}$  properties of Higgs boson can be studied independently of a production mechanism and the specific model and can be considered as the most general one.

It is generally believed that a Monte Carlo simulation of the full chain from the beam collision to detector response is the most convenient technique to investigate such studies. In our analysis, we will take as an example the  $e^+e^- \rightarrow ZH$ ;  $Z \rightarrow \mu^+\mu^-$ ;  $H \rightarrow \tau^+\tau^-$  production process. We discuss a method for the  $\mathcal{CP}$  quantum numbers measurement of the Higgs boson with a mass of 120 *GeV* for the case of  $e^+e^-$  collisions with  $\sqrt{s} = 350$  *GeV* using  $H/A^0 \rightarrow \tau^+\tau^-$ ;  $\tau^\pm \rightarrow \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ ;  $\rho^\pm \rightarrow \pi^\pm \pi^0$  decay chain. All the Monte Carlo samples have been generated with the TAUOLA Monte Carlo library [21–23]. For the production of the  $\tau$  lepton pairs the Monte Carlo program PYTHIA 6.1 is used [24]. The effects of initial state bremsstrahlung were included in the PYTHIA generation. For the  $\tau$  lepton pair decay with full spin effects included in the  $H \rightarrow \tau^+\tau^-$ ;  $\tau^\pm \rightarrow \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ ;  $\rho^\pm \rightarrow \pi^\pm \pi^0$  chain, the interface explained in Refs. [9, 10] was used. It is an extended version of the standard universal interface presented in Ref. [25], see also Ref. [26].

### 3. Mixed scalar–pseudoscalar case in Monte Carlo algorithm

Let us now, only very briefly repeat the basic information about the spin correlations and their implementation in our Monte Carlo algorithm. The detailed description of the method can be found in Ref. [21] and the full description of the algorithm is given in Ref. [9].

The main spin weight of our algorithm for generating the physical process of  $\tau$  lepton pair production in Higgs boson decay, with subsequent decay of  $\tau$  leptons as well, is given by

$$wt = \frac{1}{4} \left( 1 + \sum_{i=1}^3 \sum_{j=1}^3 R_{ij} h_1^i h_2^j \right), \quad (1)$$

where  $h_1$  and  $h_2$  are the polarimeter vectors that depend respectively on  $\tau^\pm$  decay products momenta;  $R_{ij}$  is the spin density matrix. For the mixed scalar–pseudoscalar case, when the general Higgs boson Yukawa coupling to the  $\tau$  lepton

$$\bar{\tau}(a + ib\gamma_5)\tau \quad (2)$$

is assumed, we get the following non-zero components of  $R_{ij}$ , see Ref. [13]:

$$R_{33} = -1, \quad R_{11} = R_{22} = \frac{a^2\beta^2 - b^2}{a^2\beta^2 + b^2}, \quad R_{12} = -R_{21} = \frac{2ab\beta}{a^2\beta^2 + b^2}, \quad (3)$$

where  $\beta = \sqrt{1 - 4m_\tau^2/m_H^2}$ . The crucial point is that, in general,  $a$  and  $b$  are of comparable magnitude in a  $\mathcal{CP}$  violating extension of  $\mathcal{SM}$ . For a  $\mathcal{CP}$  conserving Higgs sector, either  $a = 0$  or  $b = 0$  implying a pure pseudoscalar or scalar case respectively. For a  $\mathcal{CP}$  mixed eigenstate, both  $a$  and  $b$  are non zero. Thus any significant deviation of  $R_{12}$  or equivalently  $R_{21}$  from zero provides a signature for  $\mathcal{CP}$  violation in the Higgs sector independent of the specific model. If we express Eq. (2) with the help of the scalar–pseudoscalar mixing angle  $\phi$ :

$$\bar{\tau}N(\cos\phi + i\sin\phi\gamma_5)\tau, \quad (4)$$

the components of the spin density matrix can be expressed in the following way:

$$R_{11} = R_{22} = \frac{\cos^2\phi\beta^2 - \sin^2\phi^2}{\cos^2\phi\beta^2 + \sin^2\phi^2}, \quad R_{12} = -R_{21} = \frac{2\cos\phi\sin\phi\beta}{\cos^2\phi\beta^2 + \sin^2\phi^2}. \quad (5)$$

In the limit  $\beta \rightarrow 1$  these expressions reduce to the components of the rotation matrix for the rotation around the  $z$  axis by an angle  $-2\phi$ :

$$R_{11} = R_{22} = \cos 2\phi, \quad R_{12} = -R_{21} = \sin 2\phi. \quad (6)$$

#### 4. Definition of the observable

Let us now recall a observable which we have introduced to distinguish between scalar–pseudoscalar mixed state of the Higgs boson. The method relies on measuring the acoplanarity angle of the two planes, spanned on  $\rho^\pm$  decay products and defined in the  $\rho^+\rho^-$  pair rest frame. For that purpose the four-momenta of  $\pi^\pm$  and  $\pi^0$  need to be reconstructed and, combined, they will yield the  $\rho^\pm$  four-momenta. All reconstructed four-momenta are then boosted into the  $\rho^+\rho^-$  pair rest frame. The acoplanarity angle  $\varphi^*$ , between the planes of the  $\rho^+$  and  $\rho^-$  decay products is defined in this frame. The correlation, in the case of the Higgs boson of combined scalar and pseudoscalar couplings with the mixing angle  $\phi$ , is between transverse components of  $\tau^+$  spin polarization vector and transverse components of  $\tau^-$  polarization vector rotated by an angle  $2\phi$ . Therefore the full range of the variable  $0 < \varphi^* < 2\pi$  is of physical interest. To distinguish between the two cases  $\varphi^*$  and  $2\pi - \varphi^*$  it is sufficient, for example, to find the sign of  $p_{\pi^-} \cdot \mathbf{n}_+$ , where  $\mathbf{n}_+$  is a vector normal to the plane spanned by the visible decay products of  $\rho^+$ ,  $\mathbf{n}_+ = \mathbf{p}_{\pi^+} \times \mathbf{p}_{\pi^0}$ . The range  $0 < \varphi^* < \pi$  corresponds to the negative sign case, otherwise one should make the replacement  $\varphi^* \rightarrow 2\pi - \varphi^*$ . If no separation was made, the parity effect, in case of mixed  $H\tau\tau$  coupling, would wash itself out. Additional selection cuts need to be applied. Otherwise the acoplanarity distribution is not sensitive to transverse spin effects at all. The events need to be divided into two classes, depending on the sign of  $y_1 y_2$ , where

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} ; \quad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}. \quad (7)$$

The energies of  $\pi^\pm, \pi^0$  are to be taken in the respective  $\tau^\pm$  rest frames. In Refs. [10, 11] the methods of reconstruction of the replacement  $\tau^\pm$  rest frames were proposed with and without the help of the  $\tau$  impact parameter. We will use these methods here as well, without any modification.

#### 5. Numerical results

Let us turn our attention to the measurable distributions. We have used the scalar–pseudoscalar mixing angle  $\phi = \frac{\pi}{4}$  and, as the reference, we have used the pure scalar case  $\phi = 0$ . In our study, that is for the 350 GeV  $e^+e^-$  center-of-mass energy, Higgs boson mass of 120 GeV and Higgsstrahlung production process, we took  $N_\sigma = 62.7 \cdot 10^{-3}$  [fb] for the scale of the plot. However, in general case

$$N_\sigma = \frac{1}{4\pi} \sigma_{total}(e^+e^- \rightarrow XH) \mathcal{BR}(H \rightarrow \tau^+\tau^-) (\mathcal{BR}(\tau \rightarrow \rho\nu_\tau))^2 \quad (8)$$

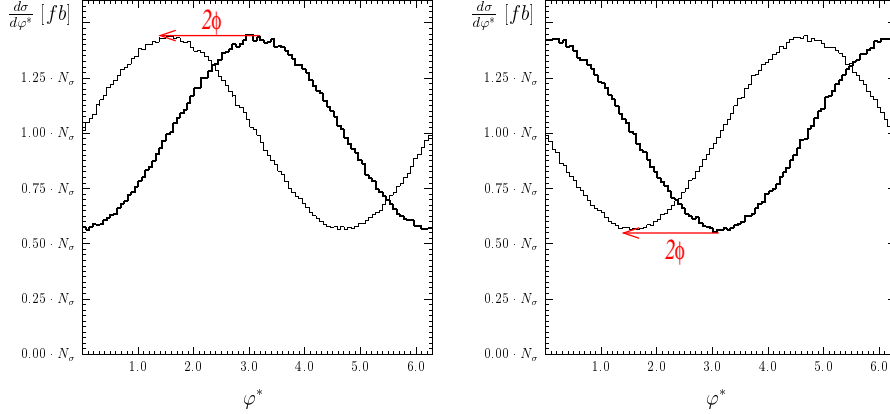


Fig. 1. The acoplanarity distribution (angle  $\varphi^*$ ) of the  $\rho^+\rho^-$  decay products in the rest frame of the  $\rho^+\rho^-$  pair. Gaussian smearing of  $\pi$ 's momenta and generator level  $\tau^\pm$  rest frames are used. The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one. The left figure contains events with  $y_1y_2 > 0$ , the right one is for  $y_1y_2 < 0$ .

is a suitable choice.

In Fig. 1 the acoplanarity distribution angle  $\varphi^*$  of the  $\rho^+\rho^-$  decay products which was defined in the rest frame of the reconstructed  $\rho^+\rho^-$  pair is shown. A detector-like set-up is included in exactly the same proportion as in Ref. [13]. Unobservable generator-level  $\tau^\pm$  rest frames are used for the calculation of selection cuts. The two plots represent events selected by the differences of  $\pi^\pm\pi^0$  energies, defined in their respective  $\tau^\pm$  rest frames. In the left plot, it is required that  $y_1y_2 > 0$ , whereas in the right one, events with  $y_1y_2 < 0$  are taken. This figure quantifies the size of the parity effect in an idealized condition, which we will attempt to approach with realistic ones.

The size of the effect was substantially diminished when a detector-like set-up was included for  $\tau^\pm$  rest frames reconstruction as well, see Fig. 2. The general shape of the distributions however remained. At the cost of introducing cuts, and thus reducing the number of accepted events, we could achieve some improvement of the method, as in Ref. [11]. If we require the signs of the reconstructed energy differences  $y_1$  and  $y_2$ , see Eq. (7), to be the same whether the method is used with or without the help of the  $\tau$  lepton impact parameter, only  $\sim 52\%$  of events are accepted. The relative size of the parity effect increases. Results are presented in Fig. 2.

Both distributions clearly distinguish between decays of scalar or mixed Higgs boson. From the measurement of these distributions we can establish

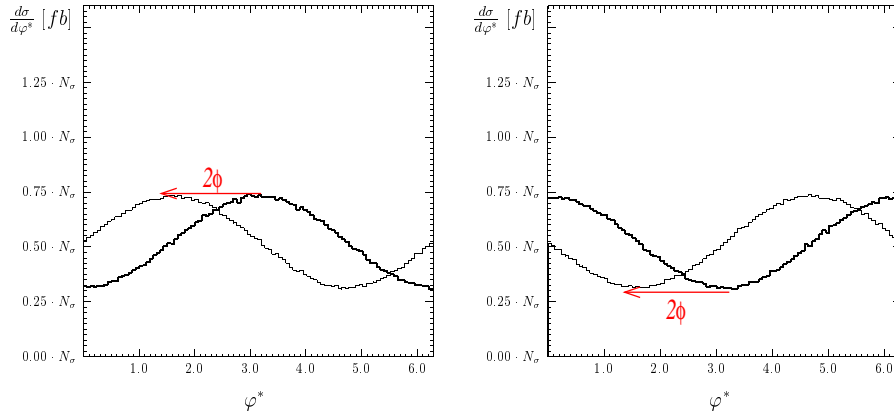


Fig. 2. The acoplanarity distribution (angle  $\varphi^*$ ) of the  $\rho^+\rho^-$  decay products in the rest frame of the  $\rho^+\rho^-$  pair. Gaussian smearing of  $\pi$ 's and Higgs boson momenta, are included. Only events where the signs of the energy differences  $y_1$  and  $y_2$  are the same, if calculated using the method without or with the help of the  $\tau$  impact parameter are taken. The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one. The left figure contains events with  $y_1 y_2 > 0$ , the right one is for  $y_1 y_2 < 0$ .

the  $\mathcal{CP}$  properties of the Higgs boson. More precisely, we have found, see [13] for details, that for an integrated luminosity of  $1 \text{ ab}^{-1}$ , at  $350 \text{ GeV}$  center-of-mass energy, a high precision  $LC$  detector such as the proposed *TESLA*, should be able to measure the scalar–pseudoscalar mixing angle for the  $H\tau\tau$  coupling with  $6^\circ$  accuracy in the case of a Standard Model Higgs boson mass of  $120 \text{ GeV}$ .

### Summary

We have studied measurement opportunities of the  $\mathcal{CP}$  properties of a  $120 \text{ GeV}$  Higgs boson at an  $e^+e^-$  collider, *e.g.* *TESLA*. Our results show that if the Higgs sector is  $\mathcal{CP}$  violating then there is a possibility to explicitly test this  $\mathcal{CP}$  violation through spin correlations between final state particles in  $H \rightarrow \tau^+\tau^-$ ;  $\tau^\pm \rightarrow \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ ;  $\rho^\pm \rightarrow \pi^\pm \pi^0$  decay chains. We have found that the mixing scalar–pseudoscalar angle can be determined with statistical precision of  $6^\circ$  for the case of  $e^+e^-$  collisions of  $350 \text{ GeV}$  center-of-mass energy with an integrated luminosity of  $1 \text{ ab}^{-1}$  and for Higgs boson mass of  $120 \text{ GeV}$ . However, we assume that the  $e^+e^- \rightarrow ZH$ ;  $Z \rightarrow \mu^+\mu^-$ ;  $H \rightarrow \tau^+\tau^-$ ;  $\tau^\pm \rightarrow \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ ;  $\rho^\pm \rightarrow \pi^\pm \pi^0$  decay chain is background free. We have not introduced any cuts *etc.* that might be required to guarantee

this.

Finally, let us note that this method can be applied to measure the parity properties of other scalar particles, not necessarily only Higgs boson(s).

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